
General Physics: Electromagnetism, Correction 7

Exercise 1 :

Consider the circuit below. The internal resistance of the battery is $r = 1 \, \Omega$, the resistance of the resistor is $R = 10 \, \Omega$. What is the *EMF* of the battery, if the heat power delivered on the resistor is $P = 40 \, \text{W}$? How much of heat energy W_B will be delivered in the battery during 1 hour?

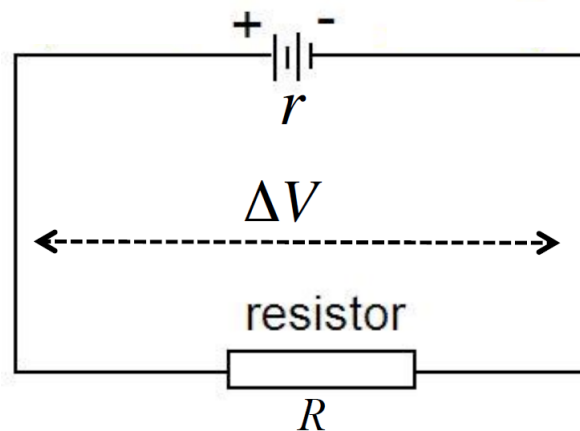


Figure 1: Simple circuit with a battery and a resistor.

Solution 1 :

The relation between the potential difference ΔV or simply V applied to the resistor and the EMF of the battery is given by:

$$\Delta V = EMF - I \cdot r \iff EMF = \Delta V + I \cdot r \quad (1)$$

where I is the current in the circuit.

For the electric power P delivered in the resistor one can write:

$$P = \frac{\Delta V^2}{R} = I^2 R \quad (2)$$

Therefore we find the following expressions for ΔV and I :

$$\Delta V = \sqrt{PR} \quad (3)$$

$$I = \sqrt{\frac{P}{R}} \quad (4)$$

Substituting ΔV and I in (8) we get:

$$EMF = \sqrt{PR} + \sqrt{\frac{P}{R}} \cdot r = \sqrt{40W \cdot 10\Omega} + \sqrt{\frac{40W}{10\Omega}} \cdot 1\Omega = 22V \quad (5)$$

The same current goes through the resistor and through the battery. Therefore, the heat delivered in the battery:

$$W_B = I^2 \cdot r \cdot t = \frac{P}{R} \cdot r \cdot t = \frac{40W}{10\Omega} \cdot 1\Omega \cdot 3600s = 14.4 \text{ kJ} \quad (6)$$

Exercise 2 :

Two 100Ω light bulbs are connected (a) in series and (b) in parallel to a 24 V battery. What is the current through each bulb in both cases? For which circuit will the bulbs be brighter?

Hint: The more power consumed, the brighter the bulb.

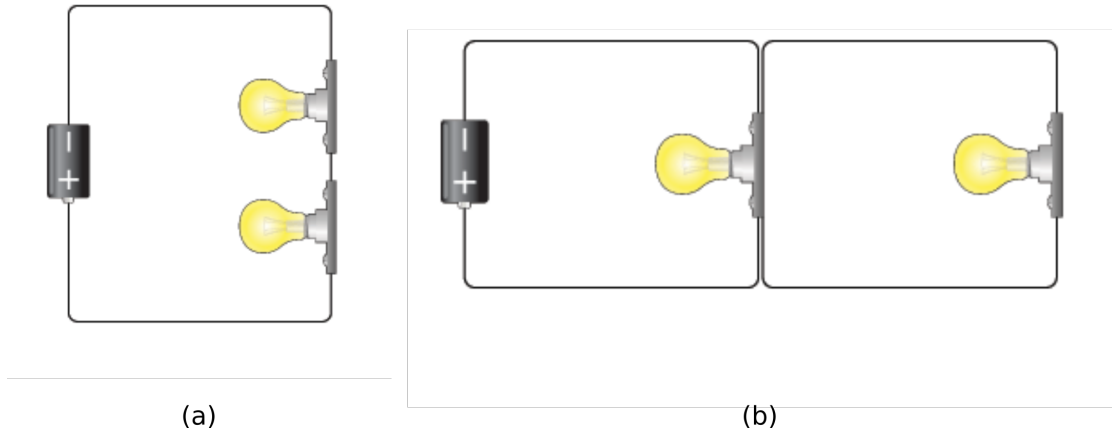


Figure 2: Series (a) and parallel (b) circuit of two light bulbs powered by a battery.

Solution 2 :

(a) In series, the total resistance is given by the sum of the resistances, so we get

$$R_{\text{eq}} = R_1 + R_2$$

$$V = IR_{\text{eq}}$$

$$V = I(R_1 + R_2)$$

$$I = V/(R_1 + R_2) = 24 \text{ V}/(100 \Omega + 100 \Omega) = 0.12 \text{ A}$$

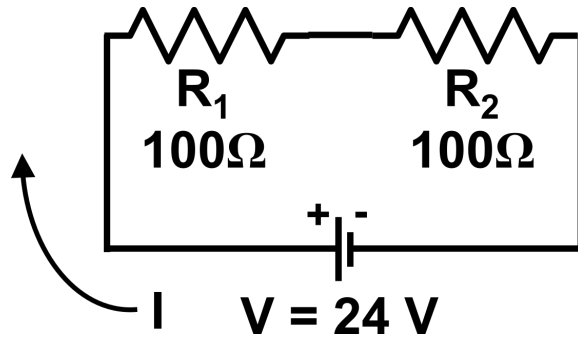


Figure 3: Schematic for the series case.

(b) In parallel, the total resistance is given by a parallel combination, so we get

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{100 \, \Omega} + \frac{1}{100 \, \Omega}$$

$$R_{\text{eq}} = 50 \, \Omega$$

$$V = IR_{\text{eq}} \Rightarrow I = V/R_{\text{eq}}$$

$$I = \frac{24 \, \text{V}}{50 \, \Omega} = 0.48 \, \text{A}$$

$$I_1 = I_2 = I/2 = 0.24 \, \text{A}$$

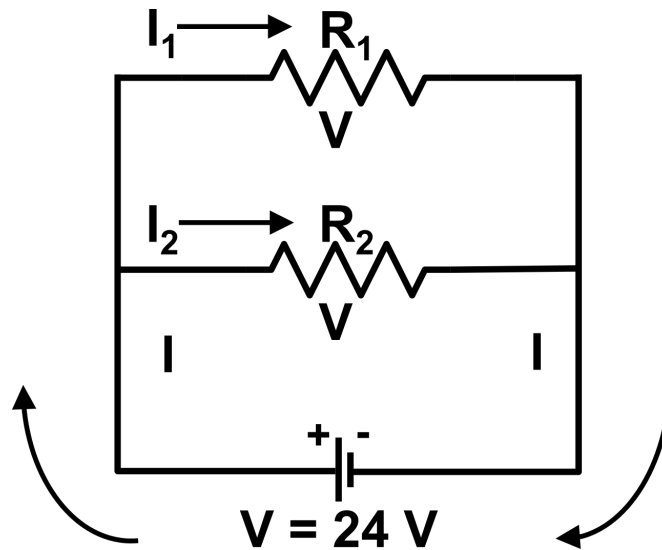


Figure 4: Schematic for the parallel case.

The power consumed by the light bulbs is given by $P = RI^2$. Therefore, $P = (0.12 \, \text{A})^2 \times 100 \, \Omega = 1.44 \, \text{W}$ in the series case and $P = (0.24 \, \text{A})^2 \times 100 \, \Omega = 5.76 \, \text{W}$ for the parallel case, so the light bulbs are brighter when connected in parallel to the battery.

Exercise 3 :

Determine the equivalent resistance of the “ladder” of equal resistors R shown in the figure below. In other words, what resistance would an *ohmmeter* read if connected between points A and B?

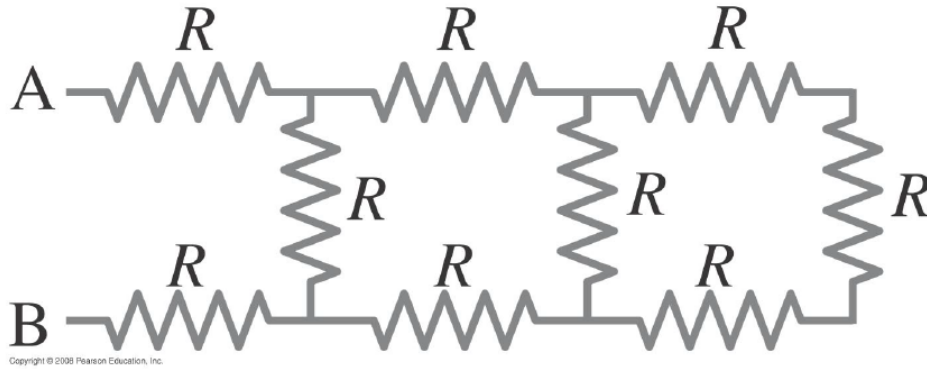
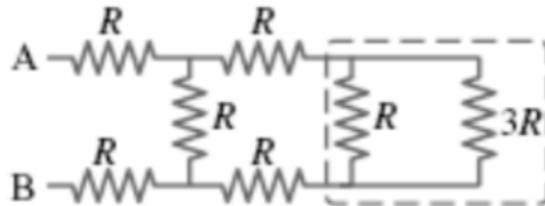


Figure 5: "Ladder" of resistors.

Solution 3 :

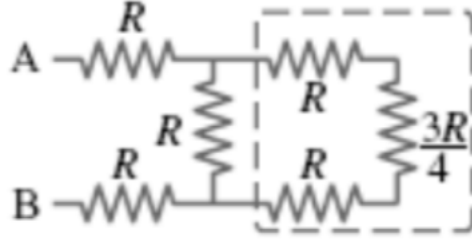
The three resistors on the far right are in series, so their equivalent resistance is $3R$. That combination is in parallel with the next resistor to the left, as shown in the dashed box in the figure below.



The equivalent resistance of the dashed box is found as follows:

$$R_{\text{eq}} = \left(\frac{1}{R} + \frac{1}{3R} \right)^{-1} = \frac{3}{4}R \quad (7)$$

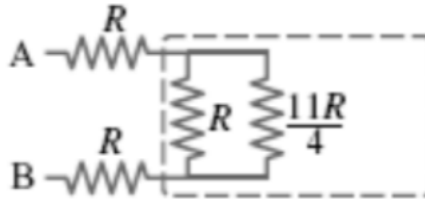
This equivalent resistance of $\frac{3}{4}R$ is in series with the next two resistors, as shown in the dashed box in the figure below.



The equivalent resistance of that dashed box is:

$$R_{eq2} = 2R + \frac{3}{4}R = \frac{11}{4}R \quad (8)$$

R_{eq2} is in parallel with the next resistor to the left, as shown in the figure below.



The equivalent resistance of that dashed box is found as follows:

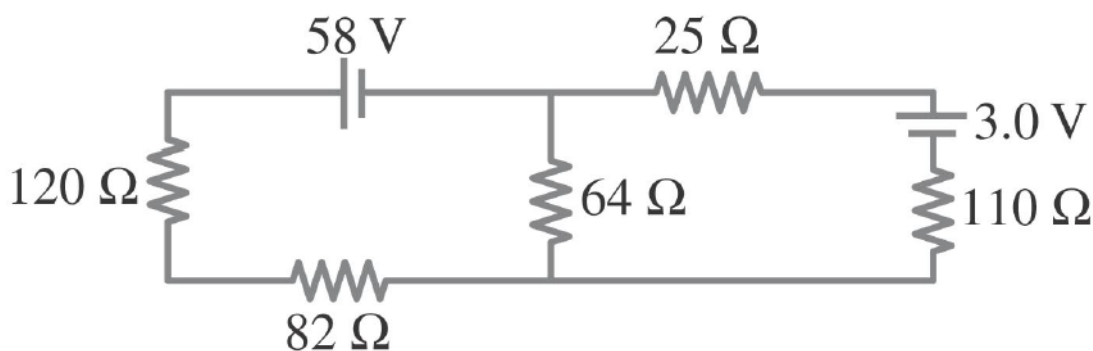
$$R_{eq3} = \left(\frac{1}{R} + \frac{4}{11R} \right)^{-1} = \frac{11}{15}R \quad (9)$$

R_{eq3} is in series with the remaining two resistors, the ones connected directly to A and B . The final resistance is given as:

$$R_{tot} = 2R + R_{eq3} = 2R + \frac{11}{15}R = \frac{41}{15}R \quad (10)$$

Exercise 4 :

Calculate the current in each resistor inside the circuit shown below.

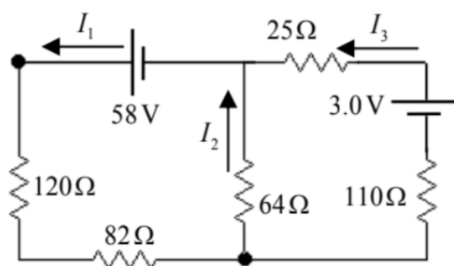


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Figure 6: Circuit with two batteries and resistors.

Solution 4 :

There are three currents involved, and so there must be three independent equations to determine those three currents.



One equation comes from Kirchhoff's junction rule applied to the junction of the three branches at the top center of the circuit.

$$I_1 = I_2 + I_3 \quad (11)$$

Another equation comes from Kirchhoff's loop rule applied to the left loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$58V - I_1 \cdot 120\Omega - I_1 \cdot 82\Omega - I_2 \cdot 64\Omega = 0 \rightarrow 202I_1 + 64I_2 = 58 \quad (12)$$

The final equation comes from Kirchhoff's loop rule applied to the right loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$3.0V - I_3 \cdot 25\Omega + I_2 \cdot 64\Omega - I_3 \cdot 110\Omega = 0 \rightarrow -64I_2 + 135I_3 = 3 \quad (13)$$

Substitute equation (25) into the left loop equation (26), so that there are two equations with two unknowns.

$$202I_1 + 64I_2 = 58 \rightarrow 202(I_2 + I_3) + 64I_2 = 58 \rightarrow 266I_2 + 202I_3 = 58 \quad (14)$$

Solve the right loop equation (27) for I_2 and substitute into the left loop equation, resulting in an equation with only one unknown, which can be solved.

$$-64I_2 + 135I_3 = 3 \rightarrow I_2 = \frac{135I_3 - 3}{64} \quad (15)$$

$$266I_2 + 202I_3 = 58 \rightarrow 266\frac{135I_3 - 3}{64} + 202I_3 = 58 \rightarrow \boxed{I_3 = 0.09235A} \quad (16)$$

$$I_2 = \frac{135I_3 - 3}{64} = \frac{135 \cdot 0.09235A - 3}{64} = \boxed{0.1479A} \quad (17)$$

$$I_1 = I_2 + I_3 = 0.1479A + 0.09235A = \boxed{0.24025A} \quad (18)$$

The current in each resistor is as follows:

$$\boxed{120\Omega: 0.24A \quad 82\Omega: 0.24A \quad 64\Omega: 0.15A \quad 25\Omega: 0.092A \quad 110\Omega: 0.092A}$$

Exercise 5 :

A good battery of a car is used to start a second car with a low battery. The good battery has an EMF of 12.5 V and an internal resistance of 0.020 Ω . Let's suppose that the low battery has an EMF of 10.1 V and an internal resistance of 0.10 Ω . The cables have a resistance $R_j = 0.0026 \Omega$ each and can be connected as shown in the figure below. Let's suppose that all the rest of the car can be represented as a $R_c = 0.15 \Omega$.

1. Find the current I_3 flowing into the starter motor if only the low battery is connected;
2. Find the current flowing into the starter motor if now also the good battery is connected.

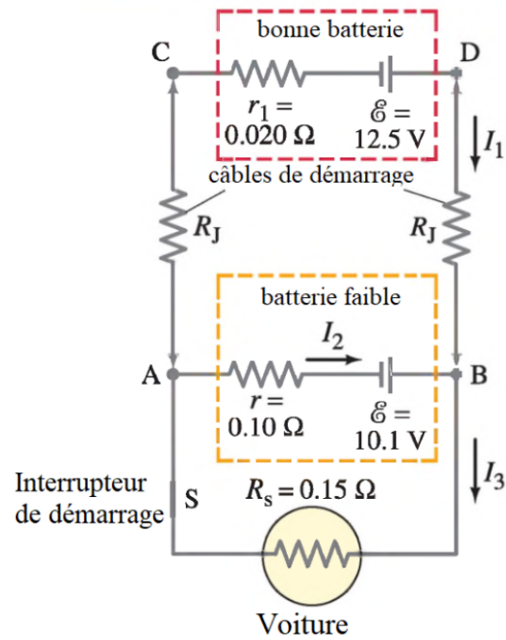


Figure 7: Schematic of the equivalent circuit.

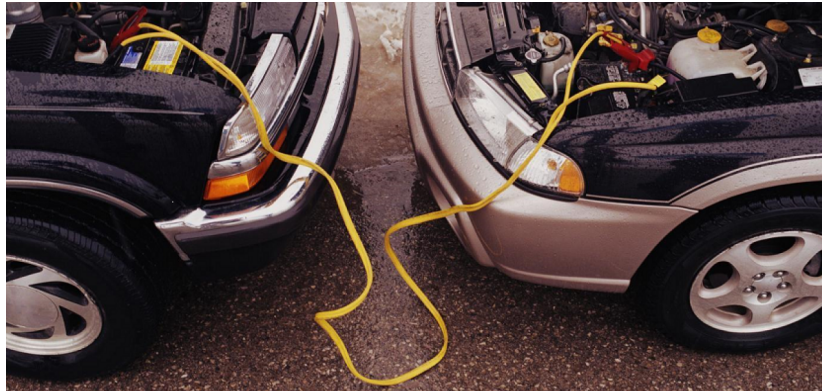


Figure 8: Two batteries, a good one and a low one, connected as shown in the circuit on top.

Solution 5 :

1. The circuit containing just the low battery and not the jumper cables is simple: one EMF of $10.1V$ connected to two resistances in series, $0.10\Omega + 0.15\Omega = 0.25\Omega$. The current is then:

$$I = \frac{V}{R} = \frac{10.1V}{0.25\Omega} = 40A \quad (19)$$

2. Apply Kirchhoff rule for the complete outer loop:

$$12.5V - I_1(2R_J + r_1) - I_3R_s = 0 \quad (20)$$

$$12.5V - I_1 \cdot 0.025\Omega - I_3 \cdot 0.15\Omega = 0 \quad (21)$$

The rule for the bottom loop with the low battery and the starter is:

$$10.1V - I_3 \cdot 0.15\Omega - I_2 \cdot 0.10\Omega = 0 \quad (22)$$

In point B we have: $I_1 + I_2 = I_3$. Using this relation, we can eliminate I_1 from equation (35) and hence we get the following system of equations:

$$12.5V - (I_3 - I_2) \cdot 0.025\Omega - I_3 \cdot 0.15\Omega = 0 \quad (23)$$

$$10.1V - I_3 \cdot 0.15\Omega - I_2 \cdot 0.10\Omega = 0 \quad (24)$$

By combining this two equations we find that $I_3 = 71A$. This current is better than what was found in 1. The values for other currents are $I_2 = -5A$ and $I_1 = 76A$. Note that I_2 is in the opposite sense than the one indicated in the figure. A circuit like this efficiently charges the low battery.

Exercise 6 :

Electronic devices (like computers for example) usually use RC circuits to protect against current failure (as shown in the Figure 1). If the power source stops working (which can be represented by opening the switch S), the capacitor will supply voltage in the circuit until it discharges. If the protective circuit has to maintain the supply voltage at at least 75% of the full voltage for 0.20 s. What is the resistance R needed to maintain this voltage? The capacity of the capacitor is $8.5 \mu F$. Suppose that the electronic device attached to the circuit consumes negligible current.

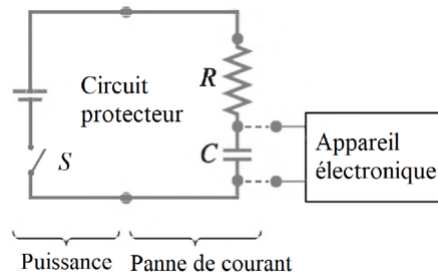


Figure 9: Scheme of an example of a protection circuit in electronic devices.

Solution 6 :

If everything works correctly, the capacitor is completely charged by the power supply. Hence, the voltage in the capacitor is the same as the voltage of the power supply and there will not be

any current going through the resistance. If there is an interruption, the voltage in the capacitor will decrease exponentially (i.e. the capacitor is discharging). We want the voltage at the terminals of the capacitor to be 75% of the full voltage after 0.20 s. The equation of the capacitor discharge is:

$$V = V_0 e^{-\frac{t}{RC}} \quad (25)$$

$$0.75V_0 = V_0 e^{-\frac{0.20 \text{ s}}{RC}} \quad (26)$$

$$0.75 = e^{-\frac{0.20 \text{ s}}{RC}} \quad (27)$$

$$\ln(0.75) = -\frac{0.20 \text{ s}}{RC} \quad (28)$$

$$R = -\frac{0.20 \text{ s}}{\ln(0.75) \cdot C} \quad (29)$$

$$(30)$$

$$R = -\frac{0.20 \text{ s}}{\ln(0.75) \cdot 8.5 \cdot 10^{-6} \text{ F}} = \boxed{81790 \Omega} \quad (31)$$

Note that the electronic device has to be directly connected to the capacitor.

Exercise 7 :

Consider a circuit below that contains several resistors $R = 5 \Omega$, two batteries with $emf = E$ each, switch S and capacitor $C = 10^{-6} \text{ F}$. At time $t = 0$ the capacitor has no charge and the switch closes.

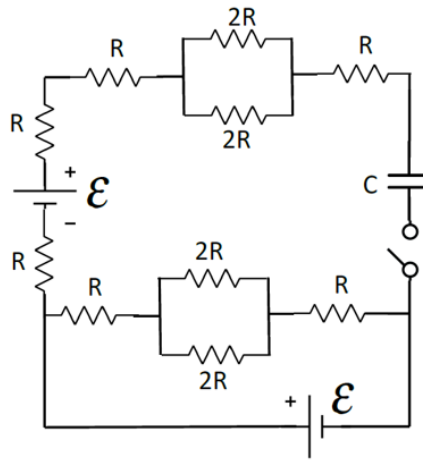


Figure 10: Circuit with two batteries, a switch and multiple resistors.

- Draw a simplified equivalent circuit of this initial complex circuit.
- Determine the time-constant for charging the capacitor (use the solution of the differential equation for a RC circuit powered by a battery from the lecture).

Solution 7 :

(a) The simplified circuit reads:

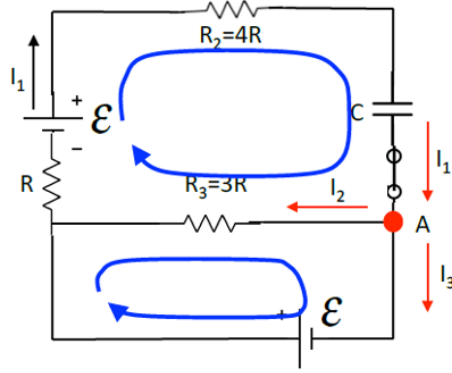


Figure 11: Simplified circuit

where R_2 was calculated the following way:

$$R_2 = R + R + \left(\frac{1}{2R} + \frac{1}{2R} \right) + R = 4R \quad (32)$$

and R_3 was calculated the following way:

$$R_3 = R + \left(\frac{1}{2R} + \frac{1}{2R} \right) + R = 3R \quad (33)$$

(b) First we apply Kirchhoff's rules and then we solve the differential equations that link charge and current on C.

1) Consider the two loops as shown in Figure 11. The choice is arbitrary, but this coincides with positive potentials of both batteries which is more convenient.

2) Define the directions of the currents in the upper loop. Here it is taken in the direction of the loop. At the junction point A the current I_1 splits into I_2 and I_3 . The direction of the current I_3 is therefore defined. It is equally possible to consider the lower loop or the second junction point.

3) Kirchhoff's rules are applied for the junction A and the two loops. Remember, the **current** is **positive**, if it is in the **same direction** as the **loop**.

$$I_1 = I_2 + I_3 \quad (34)$$

$$E = I_1 R_2 + Q/C + I_2 R_3 + I_1 R \quad (\text{upper loop}) \quad (35)$$

$$E = -I_2 R_3 \quad (\text{lower loop}) \quad (36)$$

The equation for charging the capacitor is:

$$\frac{dQ}{dt} = I_1 \quad (37)$$

4) Solving the system of equations:

Plug in (37) and (36) into (35):

$$E = \frac{dQ}{dt} \cdot (R_2 + R) + \frac{Q}{C} - E \longrightarrow \frac{dQ}{dt} \cdot 5RC + Q = 2EC \quad (38)$$

$$\frac{dQ}{dt} \cdot 5RC = 2EC - Q \quad (39)$$

After solving the differential equation (39) which is the same type of equation as for a trivial RC circuit:

$$\frac{dQ}{dt} \cdot rc = \varepsilon c - Q \longrightarrow Q = \varepsilon \cdot c \cdot (1 - e^{-\frac{t}{rc}}) \quad (40)$$

Here $r = 5R$ and $\varepsilon = 2E$. Therefore:

$$Q = 2EC \cdot (1 - e^{-\frac{t}{5RC}}) \quad (41)$$

Finally, the time constant τ for charging the capacitor is:

$$\tau = 5RC = 5 \cdot 5\Omega \cdot 10^{-6}F = 25 \cdot 10^{-6} s = 25 \mu s \quad (42)$$